

42 [9].—THOMAS R. PARKIN & DANIEL SHANKS, *Three Tables Concerning the Parity of the Partition Numbers $p(n)$ for $n < 2040000$* , Aerospace Corporation, Los Angeles, California, 1967, 398 pages of computer output bound in stiff covers and deposited in the UMT file.

Three tables computed for our paper [1] are here deposited in the UMT file. Table 1 (238 pages) extends the octal number $m/2$ of [1, Table 1] to $n = 2039999$ and thereby contains the parity of $p(n)$ to that limit in n .

Table 2 (56 pages) includes Tables 2 and 4 of [1] and lists the octuple counts from 0 to n with

$$n = r \cdot 10^s - 1$$

for $r = 1(1)9, s = 1(1)4$ and $r = 1(1)20, s = 5$. As described in [1], the k -tuple counts, $k = 2(1)7$, can be determined from these.

Table 3 (104 pages) concerns the equinumerosity of odd and even $p(n)$. It has finer detail than the Table 7 of [1] in that it lists every $n = 1000(1000)2040000$ together with every n where "Odds" = "Evens". It also includes $\max|\text{Odds-Evens}|$ in each interval here.

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1. THOMAS R. PARKIN & DANIEL SHANKS, "On the distribution of parity in the partition function," *Math. Comp.*, v. 21, 1967, pp. 466–480.

43 [9].—L. PINZUR, *Tables of Dedekind Sums*, Department of Math., University of Illinois, Urbana, Ill., 1975, 527 computer sheets deposited in the UMT file.

If x is any real number, put

$$((x)) = \begin{cases} 0, & x \text{ an integer,} \\ x - [x] - \frac{1}{2}, & \text{otherwise.} \end{cases}$$

The ordinary Dedekind sum is defined for any integer h and any positive integer k by

$$s(h, k) = \sum_{n \bmod k} ((n/k))((nh/k)).$$

It is easily shown [1] that

- (a) $s(qh, qk) = s(h, k)$, for all positive integers q ,
- (b) $s(-h, k) = -s(h, k)$,
- (c) $s(h_1, k) = s(h_2, k)$, whenever $h_1 \equiv h_2 \pmod k$.

Hence, for a given positive integer k , it is only necessary to compute $s(h, k)$ for those h such that

$$(1) \quad 1 \leq h \leq k/2, \quad (h, k) = 1.$$

The value of $s(h, k)$ is a rational number whose denominator (when in lowest terms) divides $6k$ [1]. The table consists of the integers $6k s(h, k)$ for $k = 3(1)1000$. The computation was done by repeated use of the following reciprocity relation for the Dedekind sums:

$$s(h, k) + s(k, h) = -\frac{1}{4} + \frac{1}{12} \left(\frac{h}{k} + \frac{k}{h} + \frac{1}{hk} \right).$$

This relation and properties (b) and (c) above reduce the given Dedekind sum to an expression involving a new Dedekind sum with a smaller second variable. This process continues until the second variable equals 1 or 2, at which point it stops since $s(h, 1) = s(h, 2) = 0$ for all positive integers h . For a given integer k , this algorithm takes $O(\log k)$ steps.

The table could have also been computed by storing all previously computed values of $s(h, k)$ and then calling on the reciprocity theorem only once, but this would require an enormous amount of storage. The program used has the desirable feature that individual values of $s(h, k)$ can be computed for large values of k without construction of the entire table up to that point.

AUTHOR'S SUMMARY

1. H. RADEMACHER & E. GROSSWALD, *Dedekind Sums*, Carus Math. Monograph 16, Math. Assoc. Amer., 1972.

EDITORIAL NOTE. The Dedekind sums $s(h, k)$ were previously computed for $k = 2(1)100$ and all h satisfying (1) by R. Dale Shipp (*J. Res. Nat. Bur. Standards Sect. B*, v. 69, 1965, pp. 259–263). $s(h, k)$ was given there as the quotient of two relatively prime integers. It is known that $2k(3, k)s(h, k)$ is an integer, so that the numerator and denominator of $s(h, k)$ in the present table are each divisible by 3 whenever $(k, 3) = 1$. In fact no attempt is made here to present $s(h, k)$ as the quotient of two relatively prime integers. This is not a serious drawback, however, since k is at most 1000.

The table could have been shortened further, since it is known that if $hh' \equiv 1 \pmod{k}$, then $s(h, k) = s(h', k)$. This would have introduced serious formatting problems, however, since only those h such that $1 \leq h, h' \leq k/2$ would enter into consideration. It would also have introduced problems for the user.

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